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Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS – III
(THEORY OF EQUATIONS AND VECTOR ANALYSIS)

(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. State the fundamental theorem of algebra.

2. If $\alpha_1, \alpha_2, \ldots \alpha_n$ are the roots of the equation $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$ then from the equation whose roots are $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \cdots \frac{1}{\alpha_n}$.

3. Give an example of a reciprocal equation.

4. Show that the number of real positive roots of the equation $12x^7 - x^4 + 10x^3 - 28 = 0$ cannot exceed 3.

5. Find the natural domain of the vector valued function $r(t) = \ln|t-1|i + e^t j + \sqrt{t}\,k$.

6. Compute the scalar triple product $[u\ v\ w]$ where $u = 3i - 2j - 5k$, $v = i + 4j - 4k$ and $w = 3j + 2k$.

P.T.O.

7. Find the parametric equations that correspond to the vector equation

$r(t) = \cos i + (1 + 2\sin t) j$

8. Let $r(t) = t^2 i + e^t j - (2\cos \pi t) k$, find $r'(t)$.

9. When we say that a vector f is solenoidal.

10. Give the Laplace's equation.

(10 × 1 = 10 Marks)

SECTION II

Answer any **eight** questions. These questions carry **2** marks each.

11. Solve the equation $x^3 - x^2 - 8x + 12 = 0$, given that it has a double root then $f'(x) = 3x^2 - 2x - 8$.

12. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude but opposite in sign.

13. Find the number of real roots of the equation $x^4 + 4x^3 - 4x - 13 = 0$

14. Find the equation whose roots are those of $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 2.

15. Find the unit tangent vector to the curve to the graph of $r(t) = t^2 i + t^3 j$ at the point where $t = 2$.

16. If $f(x, y) = xy$, find $D_u f(1, 2)$ for the unit vector $u = \frac{\sqrt{3}}{2} i + \frac{1}{2} j$.

17. Find the divergence of the inverse square field

$$F(x, y, z) = \frac{c}{(x^2 + y^2 + z^2)^{3/2}} (xi - yj + zk)$$

18. Find the constants a, b and c so that the vector $F = (x + by + 4z)i + (2x - 3y + cz)j + (ax - y + 2z)k$ is irrotational.

19. If F is a vector having fixed direction, show that curl F is perpendicular to F.

20. Find the work done in moving a particle in the force field $F = 3x^2i + (2xz - y)j - zk$ from $t = 0$ to $t = 1$ along the curve $x = 2t^2$, $y = t$, $z = 4t^3$.

21. Evaluate the surface integral $\iint_{\sigma} x^2z^2\,dS$ where σ is the part of the cone $\sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 2$.

22. Use the divergence to find the outward flux of the vector field $F(x, y, z) = 2xi + 3yj + z^2k$ across the unit cube $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 1$, $z = 1$.

(8 × 2 = 16 Marks)

SECTION III

Answer **any six** questions. These questions carry **4** marks each.

23. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, form the equation whose roots are $\alpha + 1/\beta\gamma$, $\beta + 1/\alpha\gamma$, $\gamma + 1/\alpha\beta$.

24. If the roots of the equation $x^3 - lx^2 + mx - n = 0$ are in AP, show that $2l^3 - 9lm + 27n = 0$.

25. If α, β, γ are the roots of $x^3 + qx + r = 0$, form the equation whose roots are $(\beta + \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

26. Transform the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ into an equation lacking the second term.

27. Find parametric equation of the tangent line to the circular helix $x = \cos t$, $y = \sin t$, $z = t$ where $t = t_0$ and use that result to find parametric equations for the tangent line at the point where $t = \pi$.

28. Show that $\nabla^2 r^m = m(m+1)r^{m-2}$. Hence deduce that $\frac{1}{r}$ is a solution of the Laplace's equation $\nabla^2 \phi = 0$.

29. Find the length of the graph of $r(t) = 3\cos t\, i + 3\sin t\, j + 2t\, k$ from $t = 0$ to $t = 2\pi$.

30. Show that the helix $x = a\cos t, y = a\sin t$, $z = ct$ where $a > 0$, has constant curvature.

31. Use Green's theorem to evaluate

$$\int_C x^2 y\, dx + x\, dy.$$

Where C is the triangle with vertices (0, 0) (1, 0) and (1, 2).

(6 × 4 = 24 Marks)

SECTION IV

Answer **any two** questions. These questions carry **15** marks each.

32. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in GP.

33. Solve the cubic $x^3 - 9x - 12 = 0$ by Cardan's method.

34. A heat-seeking particle is located at the point (2,3) on a flat metal plate whose temperature is $T(x, y) = 10 - 8x^2 - 2y^2$. Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

35. Verify Stoke's theorem, when $F = (2x - y)i - yz^2 j - y^2 zk$ where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(2 × 15 = 30 Marks)
